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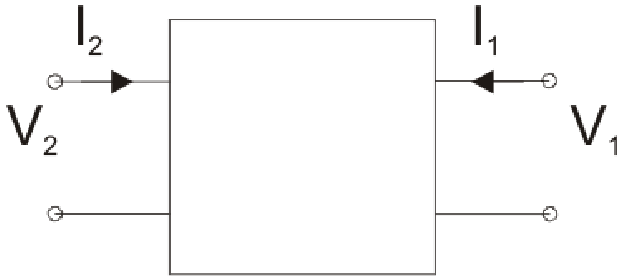
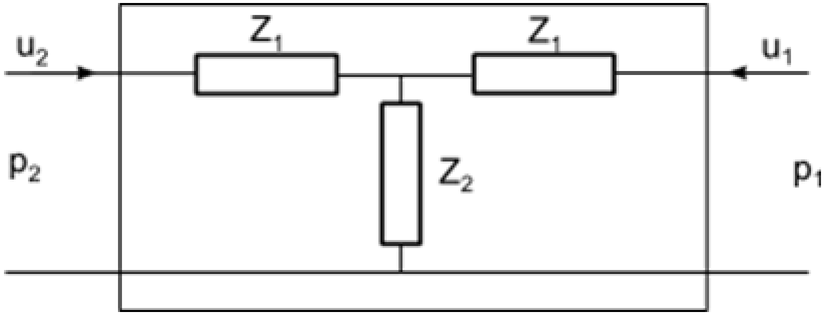
Transducer design, Part 2
MEDT8007 winter 2010

Tonni Franke Johansen

Repetition transfer matrix



$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = A \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$



Transfer matrix

$$\begin{pmatrix} p_2 \\ u_2 \end{pmatrix} = \begin{pmatrix} \cos kL & -jZ_1 \sin kL \\ \dfrac{j \sin kL}{Z_1} & -\cos kL \end{pmatrix} \begin{pmatrix} p_1 \\ u_1 \end{pmatrix}$$

Evaluation in layer m:

$$p_{m,2} = p_{m,+} e^{-jk_m z_{m,2}} + p_{m,-} e^{+jk_m z_{m,2}}$$

$$p_{m,1} = p_{m,+} e^{-jk_m z_{m,1}} + p_{m,-} e^{+jk_m z_{m,1}}$$

etc.

Piezoelectric material 1D material equations

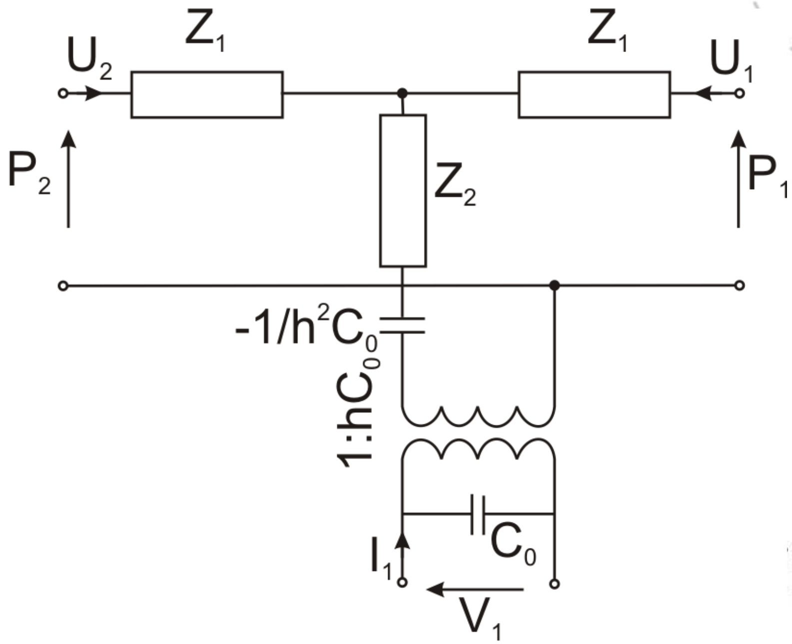
A piezo electric plate is close to a capacitance, modified with electro mechanical coupling through the piezo electric constant, h.

$$p(z,t) = -\frac{1}{\kappa}\frac{\partial \psi(z,t)}{\partial z} + hD(z,t)$$

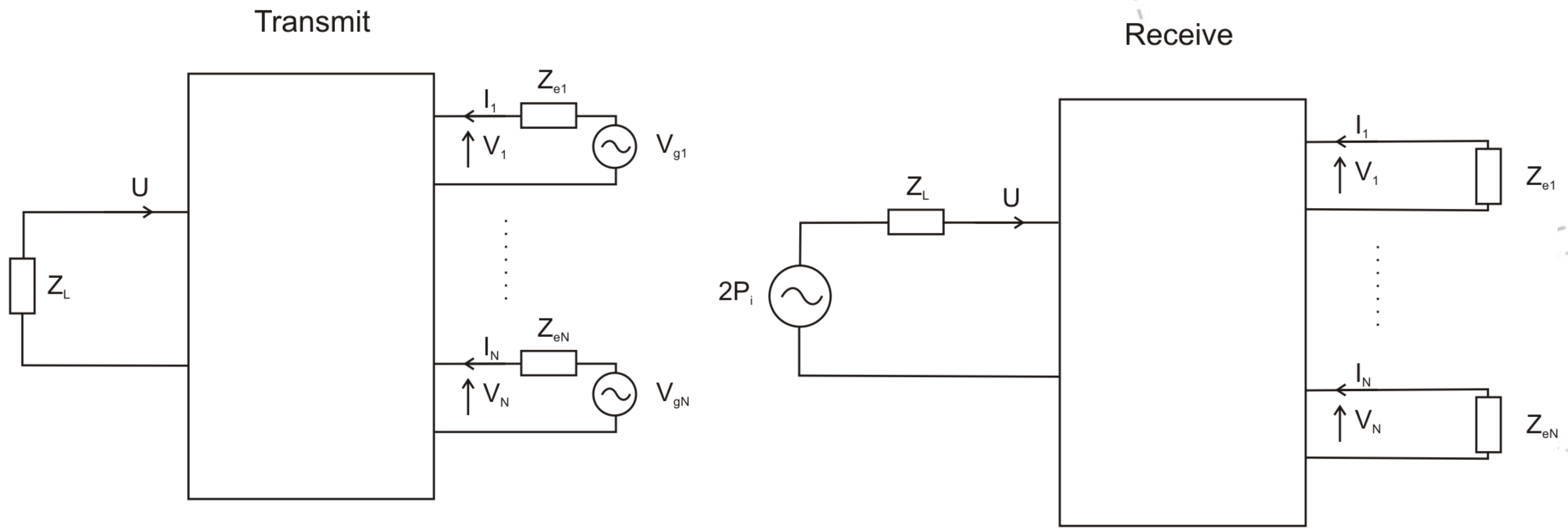
$$E(z,t) = -h\frac{\partial \psi(z,t)}{\partial z} + \frac{1}{\varepsilon}D(z,t)$$

$$\mathrm{V}(\omega) = \frac{1}{\mathrm{j}\omega \mathrm{C}_0}\mathrm{I}(\omega) + \frac{\mathrm{h}\left(\mathrm{U}(\mathrm{L},\omega) - \mathrm{U}(0,\omega)\right)}{\mathrm{j}\omega}$$

- 3 port model, (2 mechanical, 1electrical)
- transmission line describe vibration
- mechanical and electrical variables can be extracted from the material equations



Admittance matrix model (xTrans)

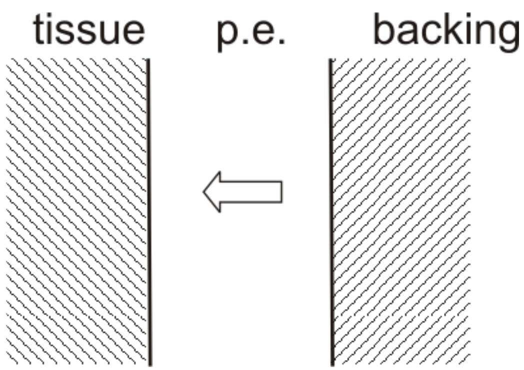


$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \\ U \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} & H_{tt,s,1} \\ \vdots & \ddots & \vdots & \vdots \\ Y_{N1} & \cdots & Y_{NN} & H_{tt,s,N} \\ H_{tt,s,1} & \cdots & H_{tt,s,N} & Y_m \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \\ 2P_i \end{bmatrix}$$

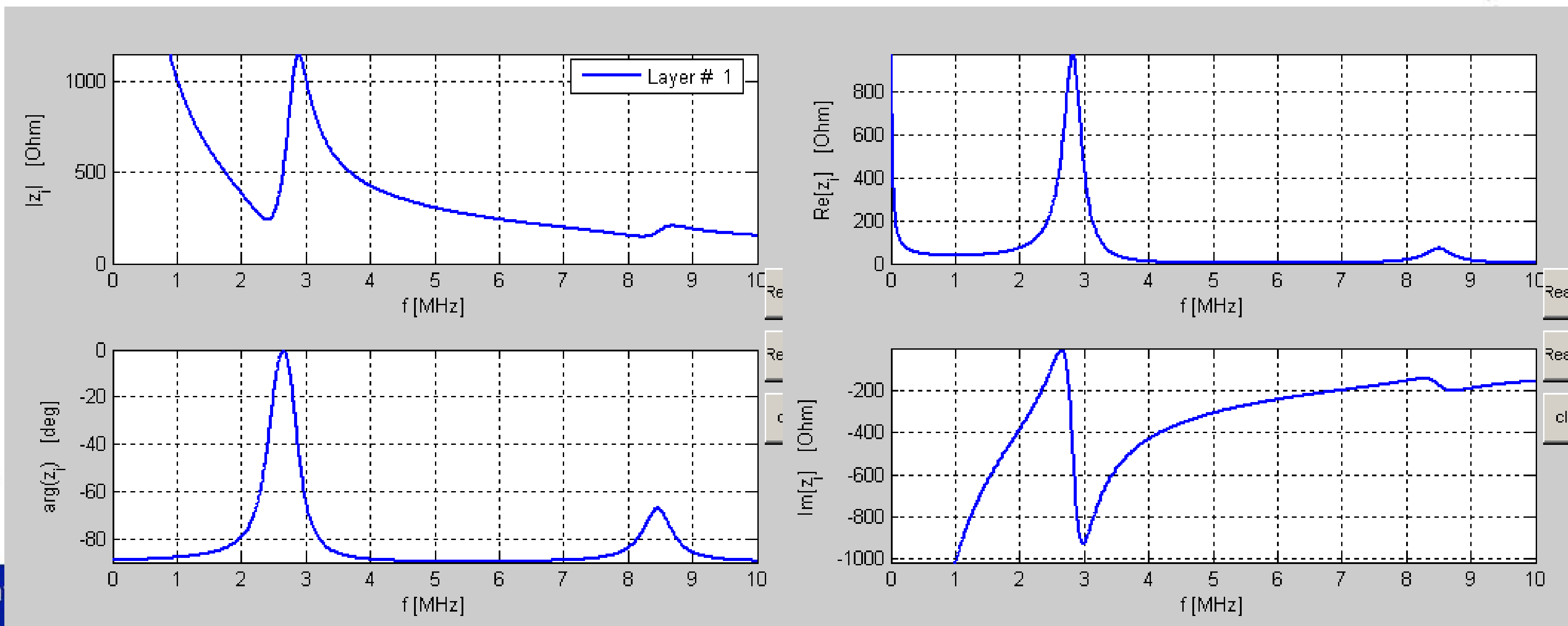
$$H_{tt,s,n} = \frac{U}{V_n} \quad , \quad V_i = 0, i \neq n$$

$$H_{tt,s,n} = \frac{I_n}{2P_i} \quad , \quad V_i = 0, i \in [1, N]$$

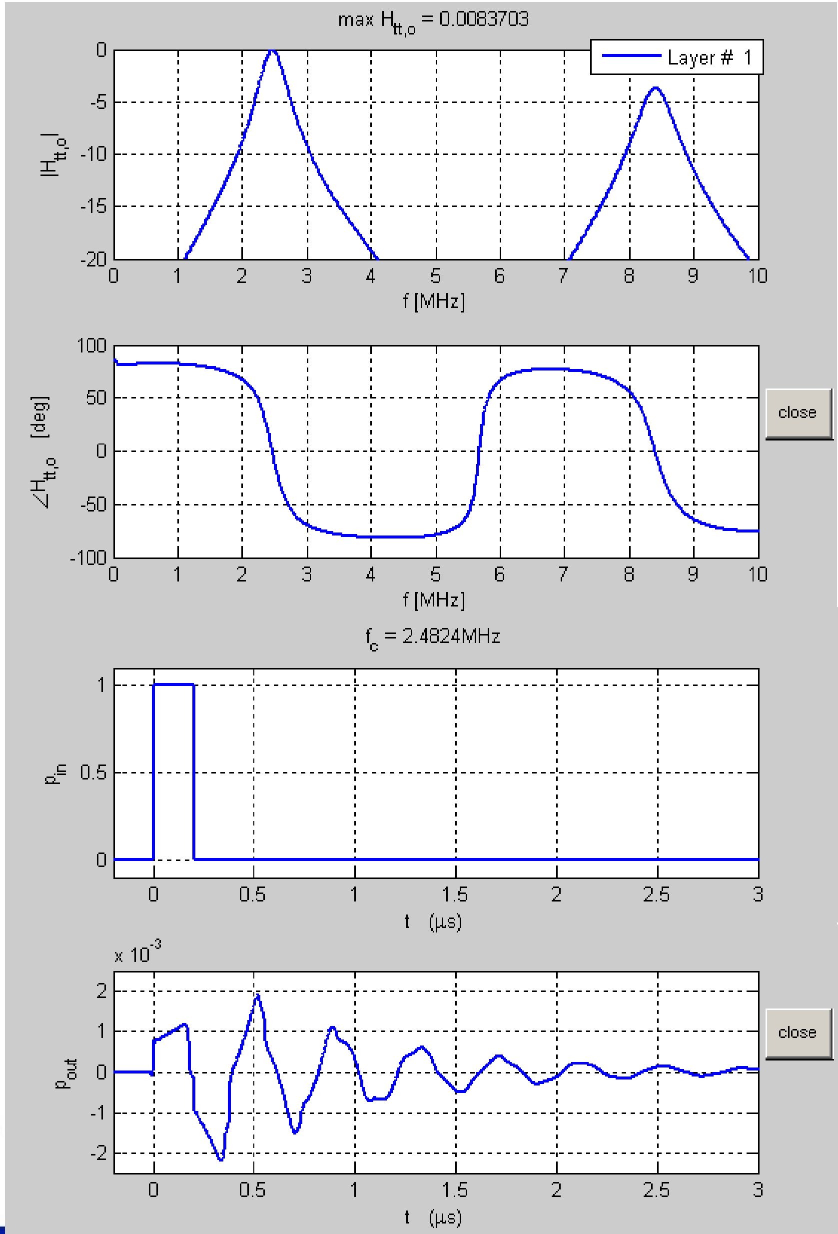
Ex. Pz29 no matching



	h 10^8 V/m	$\varepsilon/\varepsilon_0$	Z MRayl	c m/s	L mm
backing			3		inf
p.e.	19.6	1220	33.6	4440	0.78
tissue			1.65		inf



Ex. Pz29 no matching, cont.



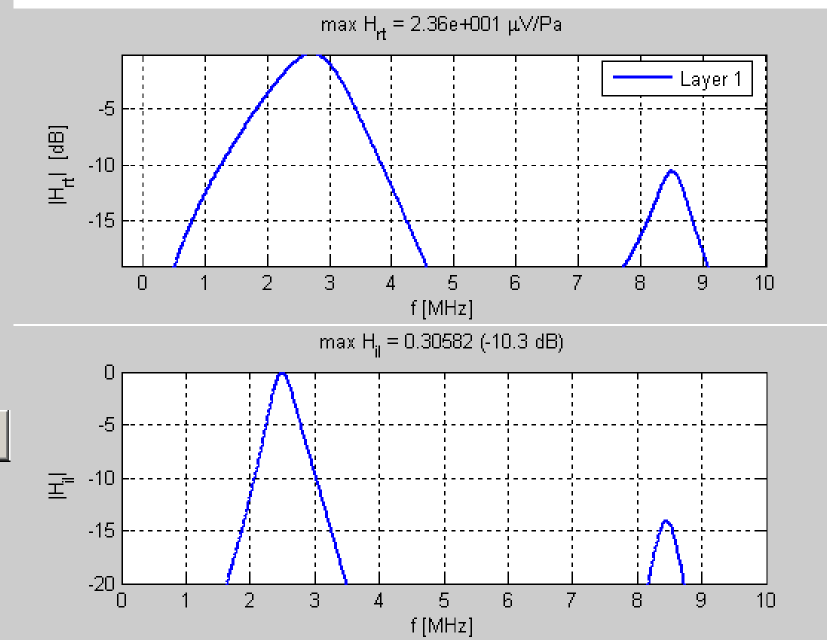
$Bw_{tx,-3dB}$ = 16%

$Bw_{rx,-3dB}$ = 44%

$Bw_{2w,-6dB}$ = 24%

receive load:

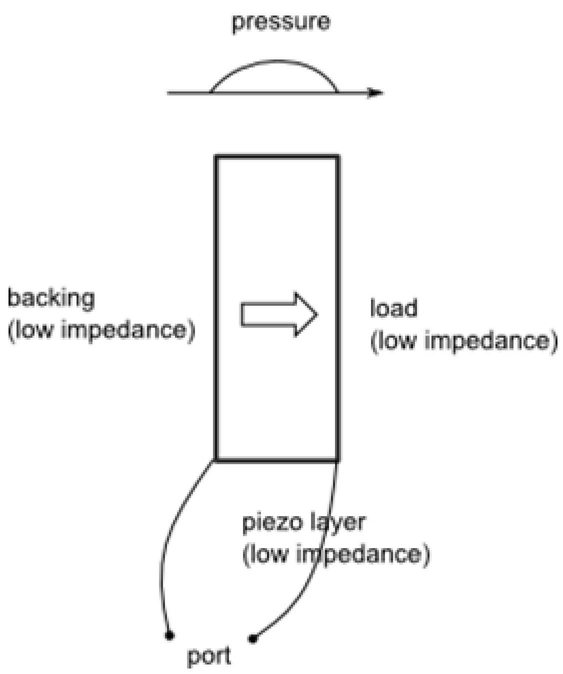
500 Ω – 20nH



Overview

- Transducer function
  - Thickness of piezo electric layer
  - Acoustical matching
  - Piezoelectrical composites/efficient material parameters
- Interaction between the transducer and electrical signal chain
  - Simple tuning
  - Influence of the cable
- How to use process further with results from xTrans
  - Pulse echo response
- Finite lateral dimensions

Thickness of the piezo electric plate

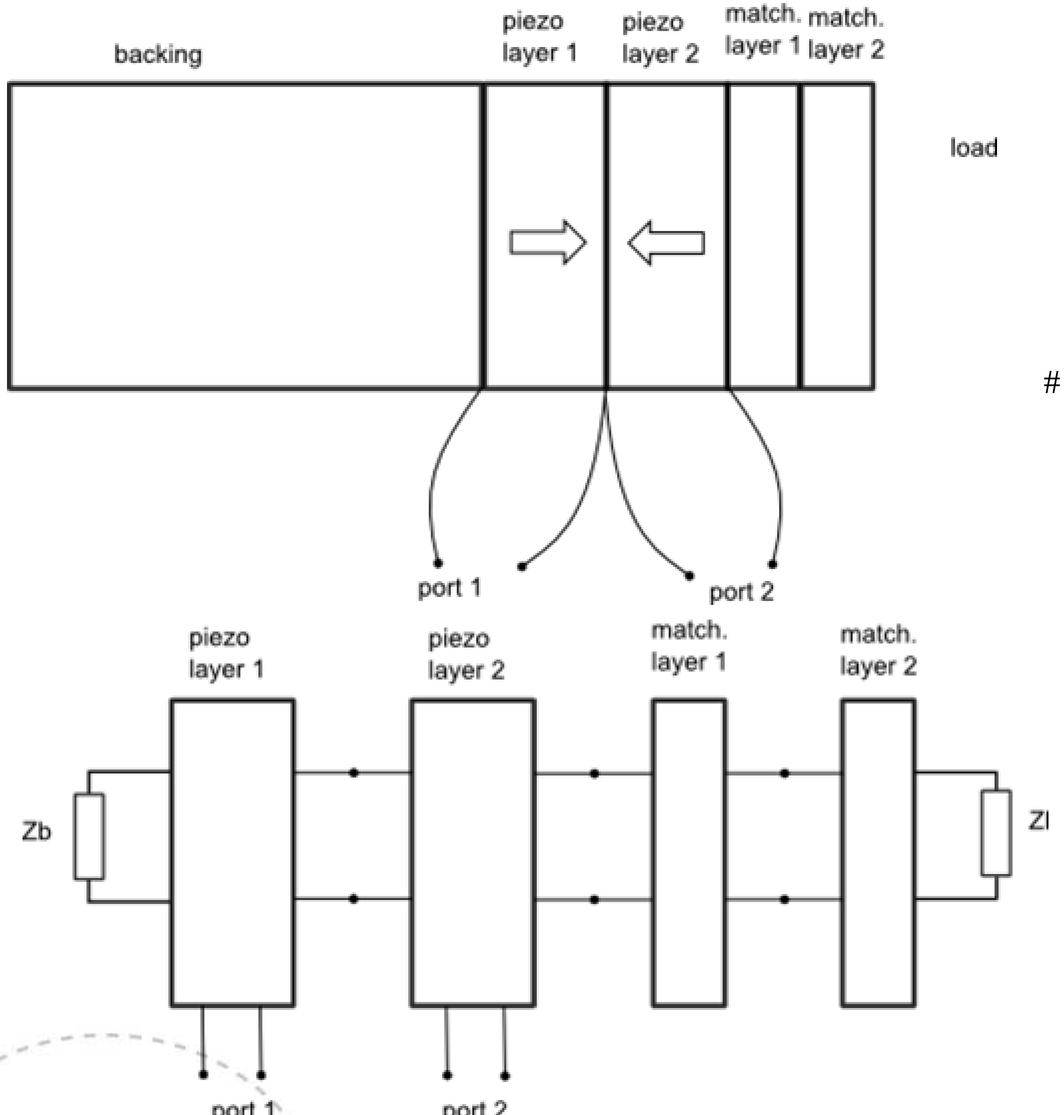


Low impedance at both side of plate indicate resonance at plate thickness $n\lambda/2$.
Only odd n give couples to electrical port, as for even n U(L)=U(0).

$$V(\omega) = \frac{1}{j\omega C_0} I(\omega) + \frac{h\left(U(L,\omega) - U(0,\omega)\right)}{j\omega}$$

High impedance backing indicate resonance at plate thickness $(2n-1)\lambda/4$.

Acoustic matching



Matching layer thickness: λ/4

Piezo impedance: Z_o
Load impedance: Z_L
Maximal flat response
(Angelsen, Cobbold, McKeighen)

# layers\Layer #	1	2	3
1	$Z_L^{2/3} Z_0^{1/3}$		
2	$Z_L^{3/7} Z_0^{4/7}$	$Z_L^{6/7} Z_0^{1/7}$	
3	$Z_L^{4/15} Z_0^{11/15}$	$Z_L^{10/15} Z_0^{5/15}$	$Z_L^{14/15} Z_0^{1/15}$

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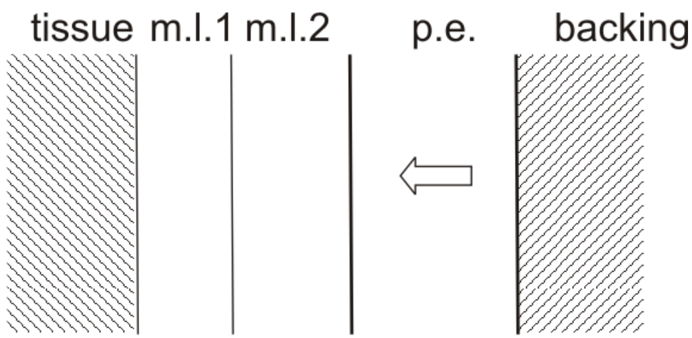
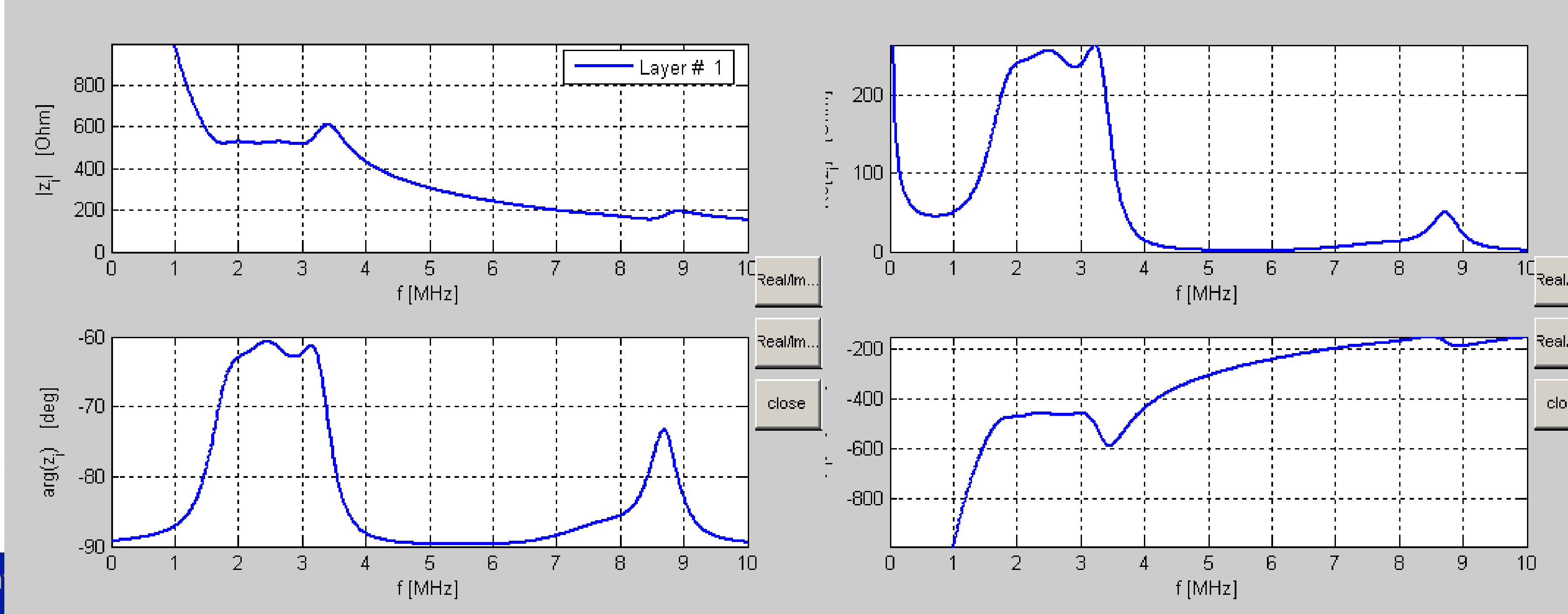
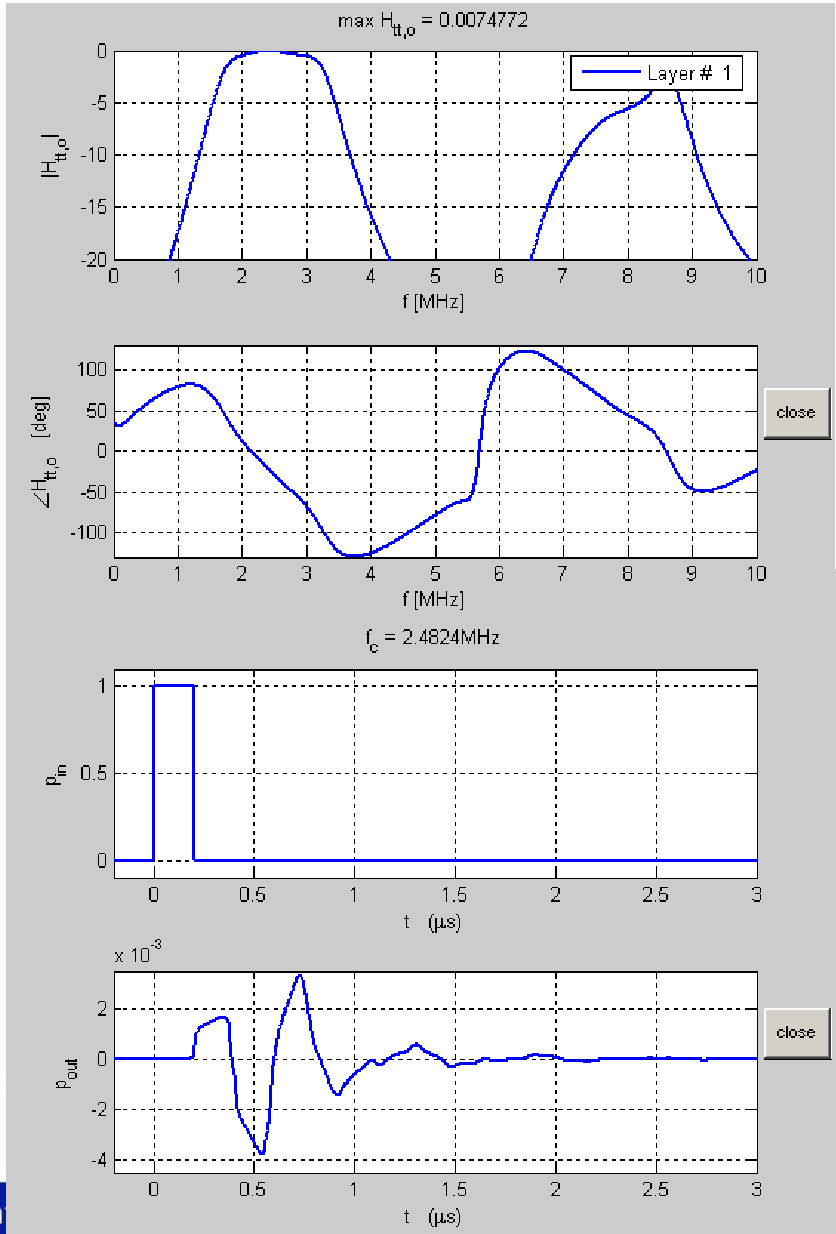
Ex.2 Pz29 with matching



	h 10^8 V/m	$\varepsilon/\varepsilon_0$	Z MRayl	c m/s	L mm
backing			3		inf
p.e.	19.6	1220	33.6	4440	0.78
m.l.2			9.2	3000	0.31
m.l.1			2.5	3000	0.31
tissue			1.65		inf



Ex.2 Pz29 with matching, cont.



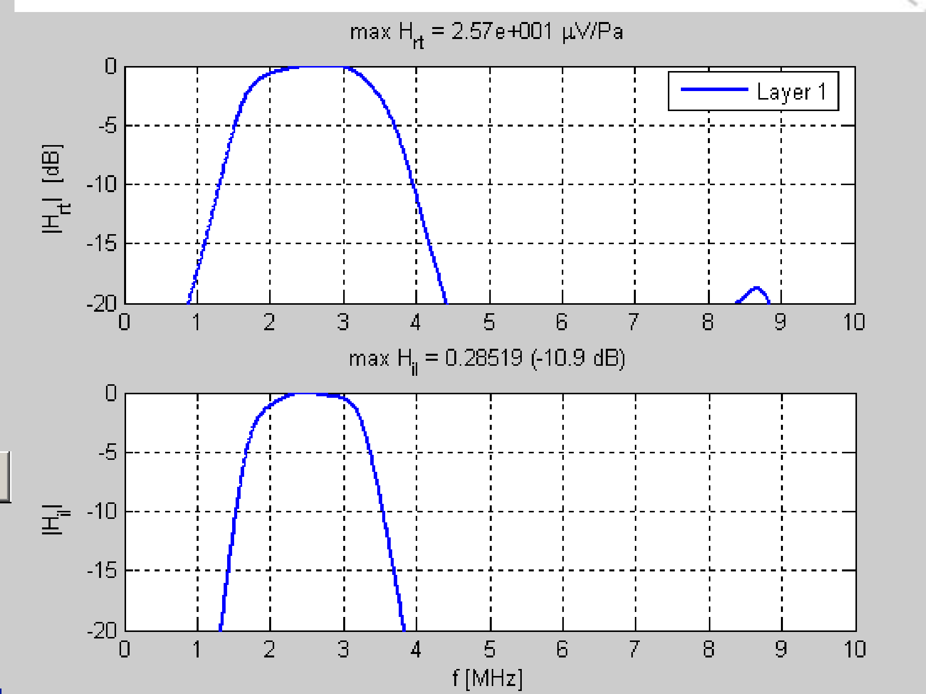
$Bw_{tx,-3dB}$ = 67%

$Bw_{rx,-3dB}$ = 74%

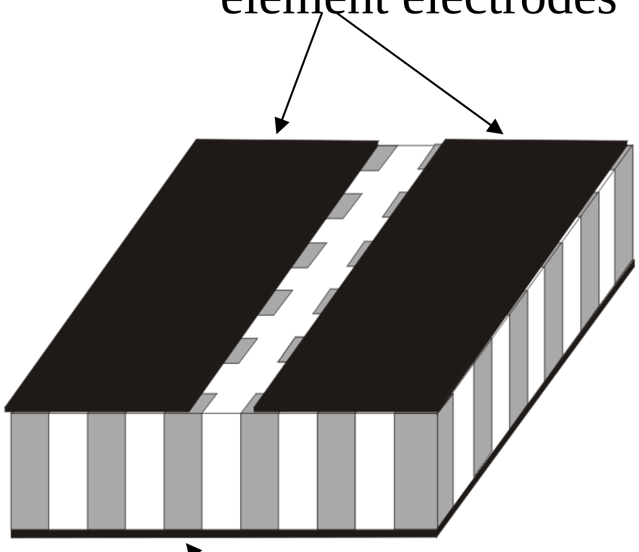
$Bw_{2w,-6dB}$ = 70%

receive load:

200 Ω – 20nH



Piezo electric composite



Piezo ceramic – diced filled with polymer

New ”equivalent material”

- better mechanical matching
- geometrical shaping
- less lateral coupling

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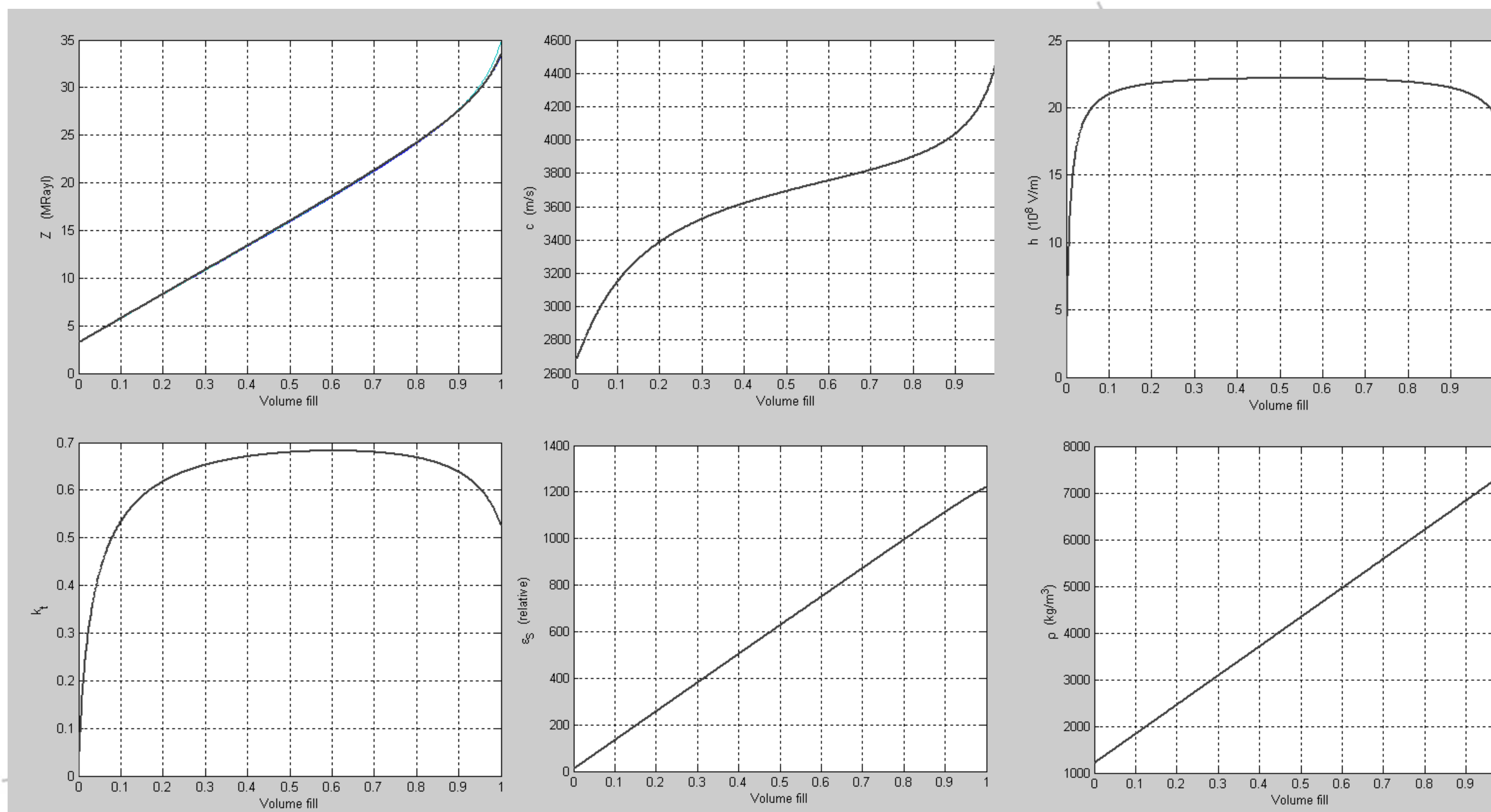
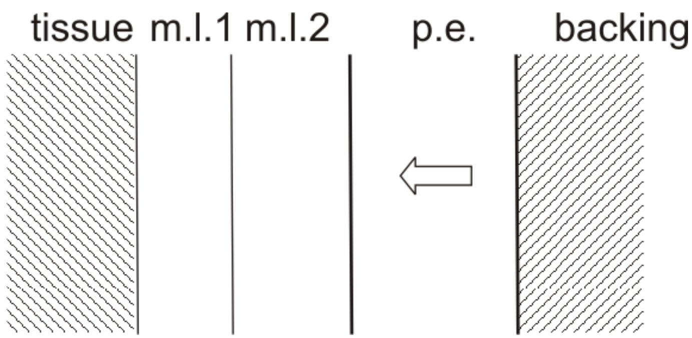
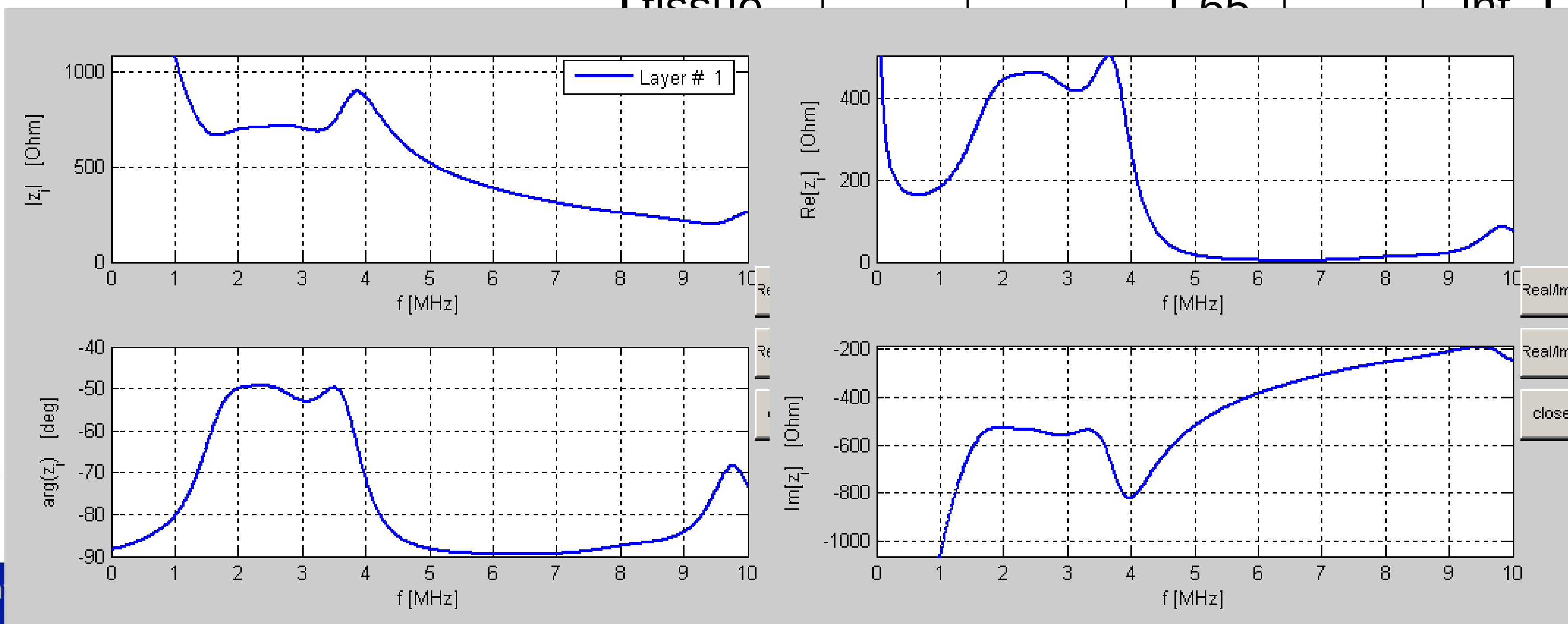
Pz29-epoxy composite



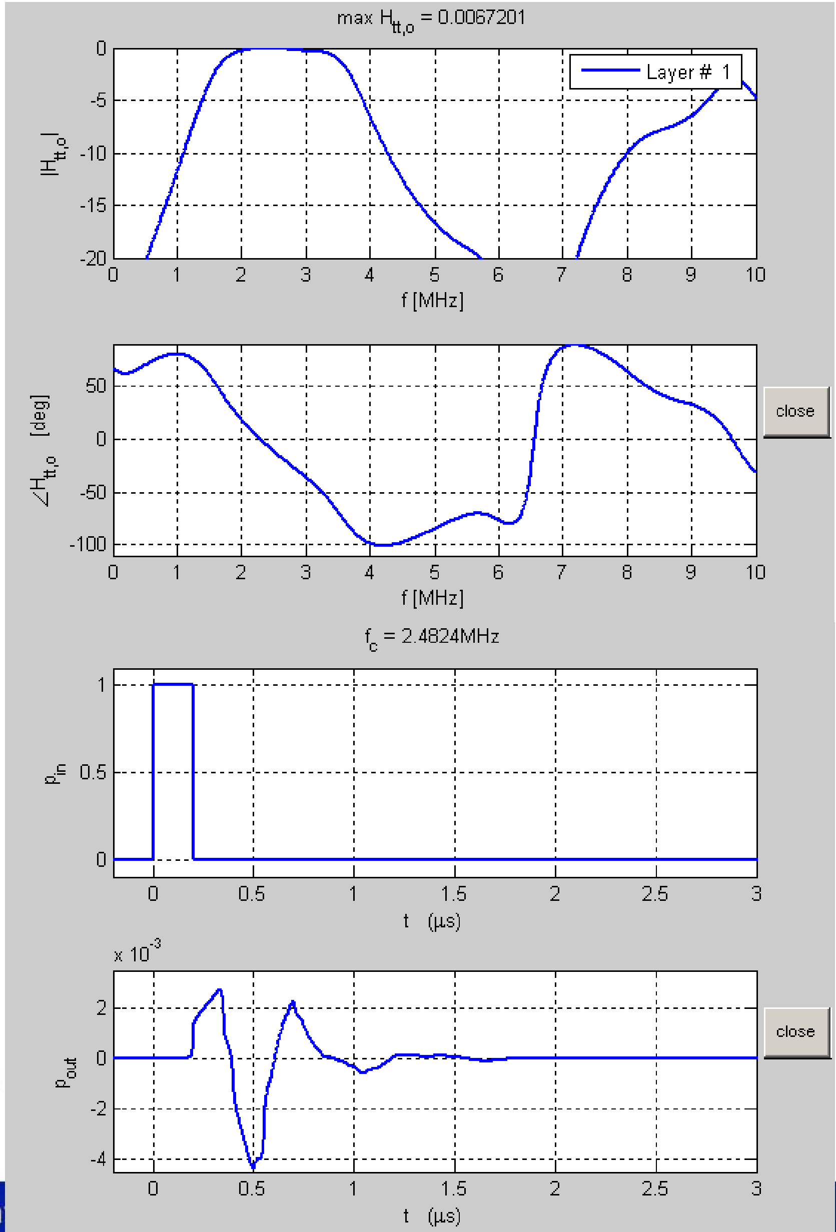
Ex.3 Pz29 composite (46%)with matching



	h 10^8 V/m	$\varepsilon/\varepsilon_0$	Z MRayl	c m/s	L mm
backing			3		inf
p.e.	22.2	580	15	3670	0.56
m.l.2			5.8	3000	0.30
m.l.1			2.3	3000	0.30
tissue			1.65		inf



Ex.3 Pz29 composite (46%)with matching, cont.



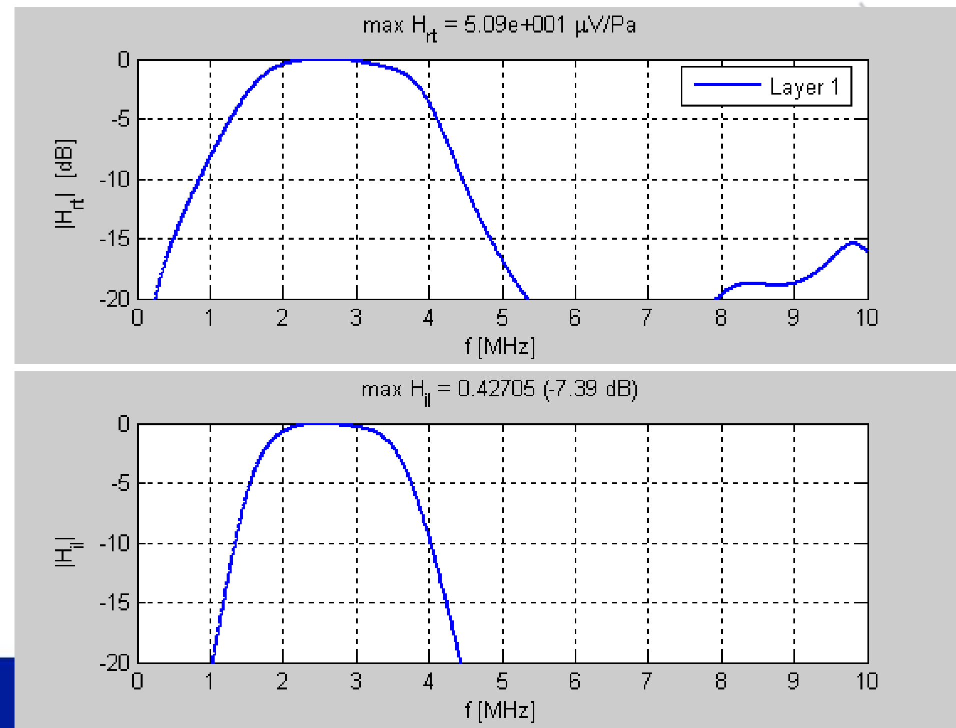
$Bw_{tx,-3dB}$ = 84%

$Bw_{rx,-3dB}$ = 90%

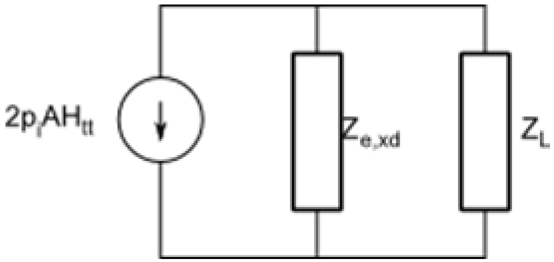
$Bw_{2w,-6dB}$ = 87%

receive load:

500 Ω – 30nH

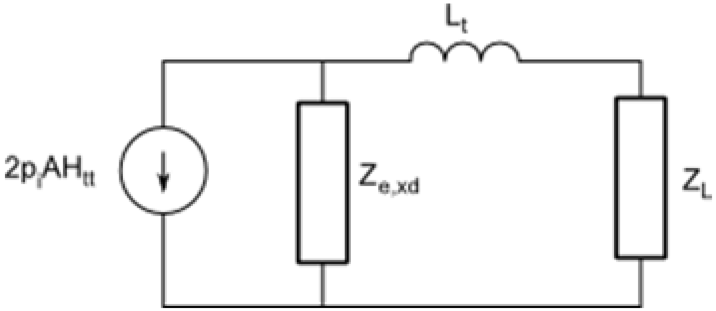


Electrical interaction



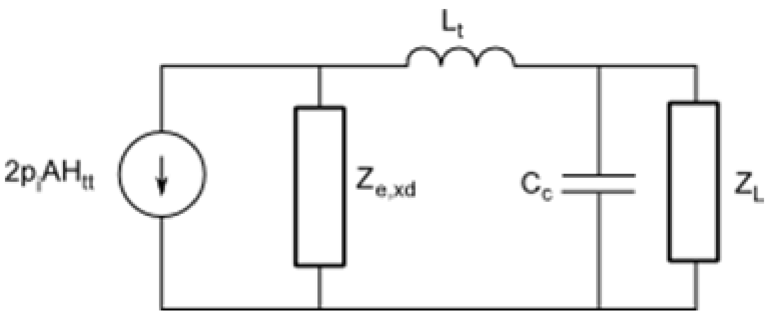
Receive sensitivity dependent on load

$$V_L = 2p_i AH_{tt} \frac{Z_{e,xd} Z_{eL}}{Z_{e,xd} + Z_{eL}}$$



Receive sensitivity, serial tuning

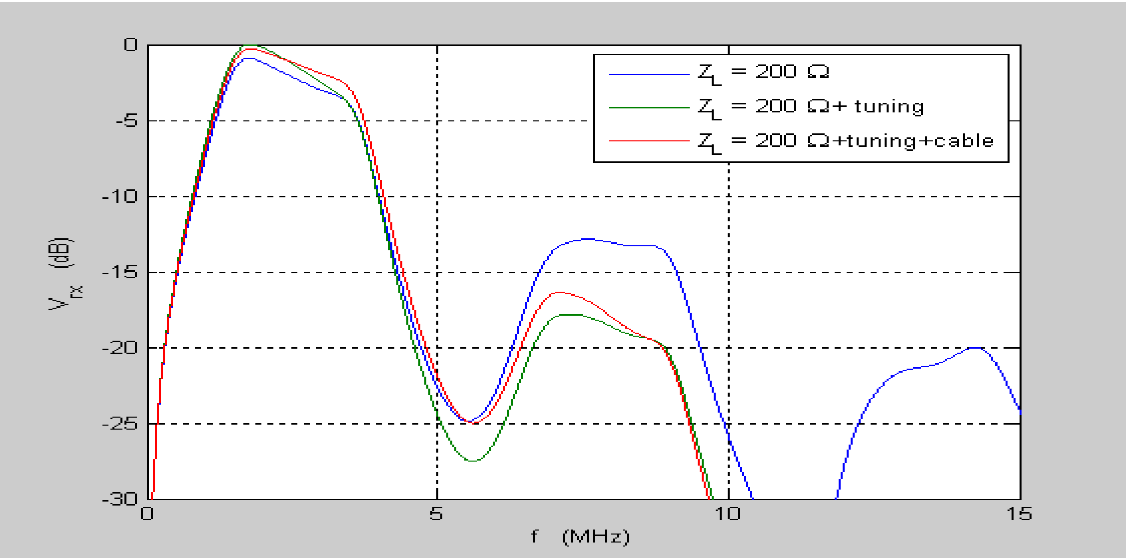
$$V_L = 2p_i AH_{tt} \frac{Z_{e,xd} Z_{eL}}{Z_{e,xd} + Z_{eL} + j\omega L_t}$$

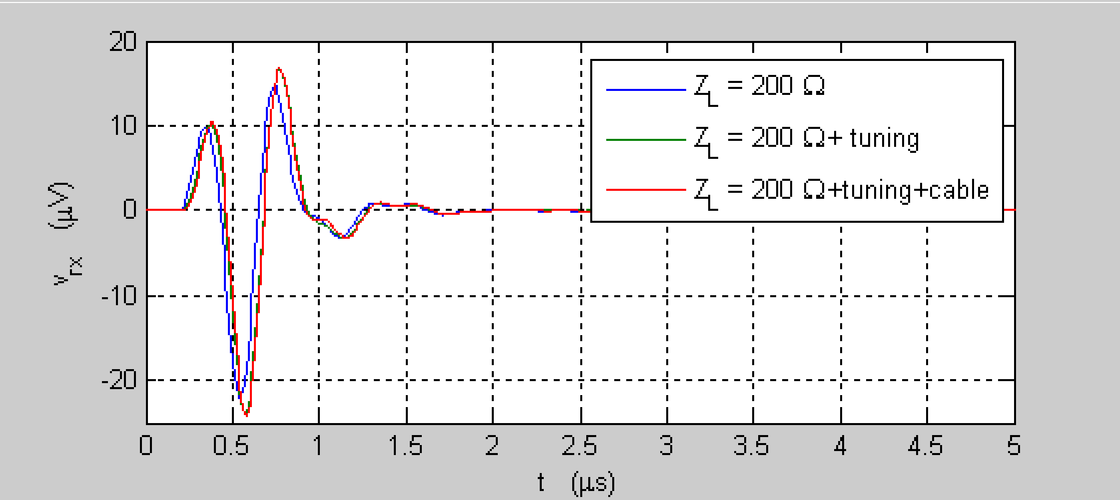


Receive sensitivity, serial tuning, low frequency cable approximation.

$$V_L = 2p_i AH_{tt} \frac{Z_{e,xd} Z_{eL}}{Z_{e,xd} + Z_{eL} + j\omega\left(L_t + C_c Z_{eL} Z_{e,xd}\right) - \omega^2 L_t C_c Z_{eL}}$$

Electrical interaction





Applied signal:
sinus 2.5 MHz, 1 periode

Tuning, 7.5 µH
Cable, 100 pF
Z_{eL}=200Ω

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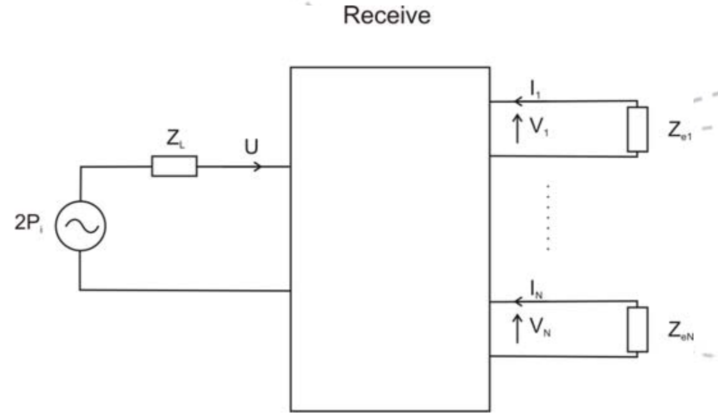
Electrical interaction - Reflection coefficients

$$R_s = \frac{P_r}{P_i} = 1 - 2Y_M Z_L$$

$$\begin{pmatrix} \mathbf{I} \\ U \end{pmatrix} = \begin{pmatrix} \mathbf{Y} & \mathbf{H_{tt}} \\ \mathbf{H_{tt}^T} & Y_M \end{pmatrix} \begin{pmatrix} \mathbf{V} \\ 2P_i \end{pmatrix}$$



$$\mathbf{I} = -\mathbf{Y_r V}$$

$$-\mathbf{Y_r V} = \mathbf{YV} + \mathbf{H_{tt}} 2P_i$$

$$\mathbf{V} = -\left(\mathbf{Y} + \mathbf{Y_r}\right)^{-1} \mathbf{H_{tt}} 2P_i$$

$$U = \left(-\mathbf{H_{tt}^T}\left(\mathbf{Y} + \mathbf{Y_r}\right)^{-1} \mathbf{H_{tt}} + Y_M\right) 2P_i$$

$$U_r = U - U_i = \left(2Y_M - 2\mathbf{H_{tt}^T}\left(\mathbf{Y} + \mathbf{Y_r}\right)^{-1} \mathbf{H_{tt}} - \frac{1}{Z_L}\right) P_i$$

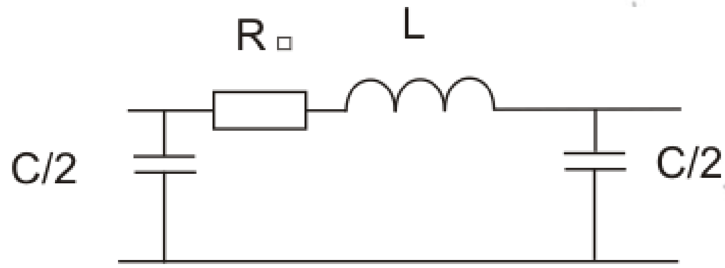
$$R_z = \left(1 - 2Y_M Z_L + 2Z_L \mathbf{H_{tt}^T}\left(\mathbf{Y} + \mathbf{Y_r}\right)^{-1} \mathbf{H_{tt}}\right)$$

$$R_z = \left(R_s + 2Z_L \mathbf{H_{tt}^T}\left(\mathbf{Y} + \mathbf{Y_r}\right)^{-1} \mathbf{H_{tt}}\right)$$

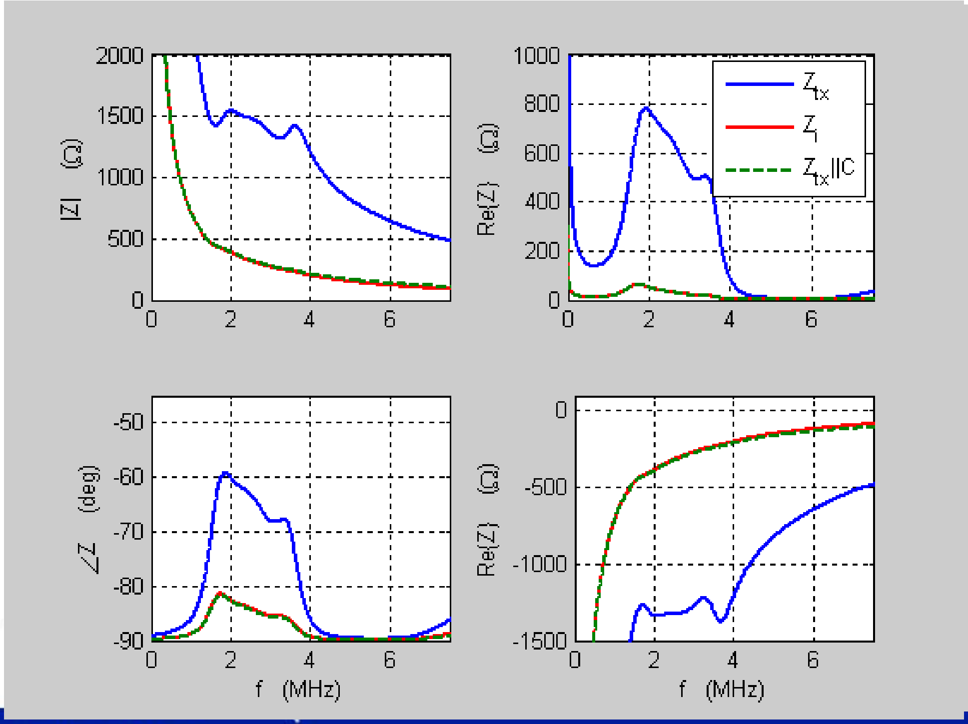
$$R_o = \left(R_s + 2Z_L \mathbf{H_{tt}^T} \mathbf{Y}^{-1} \mathbf{H_{tt}}\right)$$

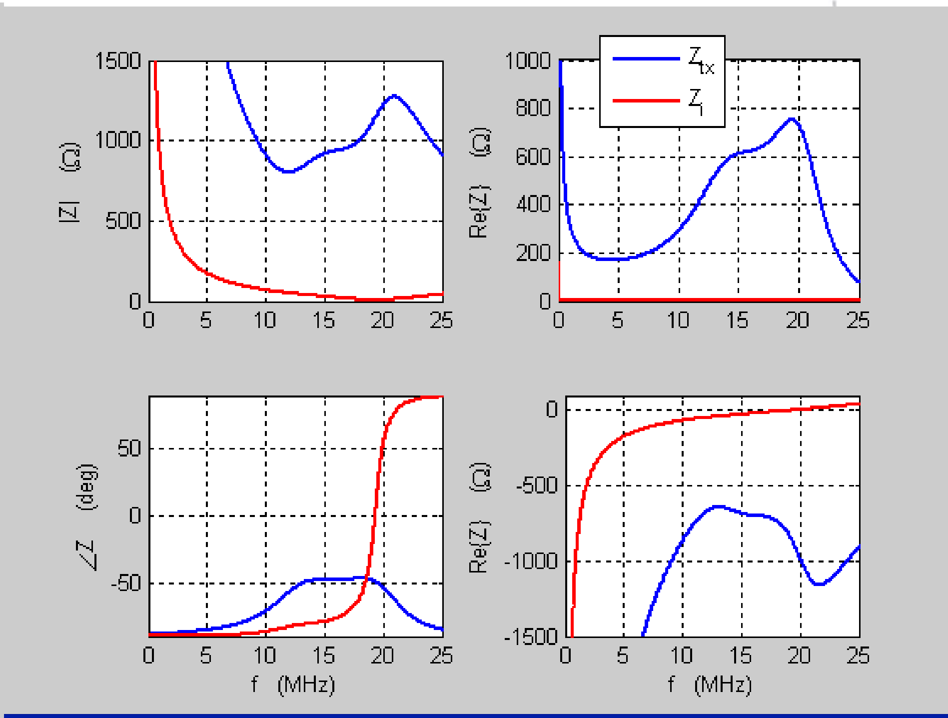
Electrical interaction - the cable

A typical cable:
L=400 nH/m
C=62 pF/m
R=2Ω/m
ℓ= 2.5m



π-model ok up to $\ell \approx \lambda/4$





Processing on xTrans results

- xTrans results in file (xduce.prev or renamed), or in workspace, sXtransParam struct.
- sXtransParam.Y_sys is the admittance matrix
- See example on previous plot in plot_Vrx.m

- Pulse echo with total reflection

$$V_L = \underbrace{V_t H_{tt} Z_L}_{\text{transmit}} \underbrace{2 A H_{tt} H_r}_{\text{receive}}$$

Finite lateral dimensions

- For a finite transducer, especially linear array elements one have substantial deviation from the plane wave assumption. However the 1D analysis give a fair first approximation.
- FEM as presented below can give a more accurate simulations (McKeighen).
- Note the wide vibration area. Effective element size and azimuth directivity is measured to characterize the array.